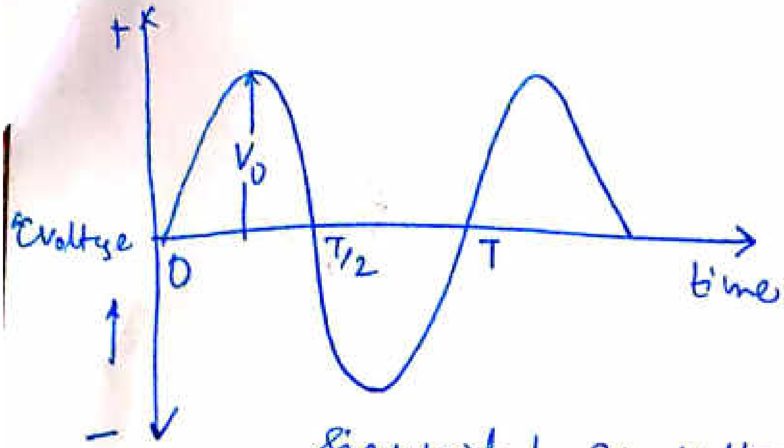
075

Alternating Currents



Sinusoidal ac voltage

passes through a complete cycle of changes both in magnitude and dir^n at regular intervals of time.

$v = V_0 \sin \omega t$

or $v = V_0 \cos \omega t$ $\quad V_0 \to$ peak value

$i = I_0 \sin(\omega t + \phi)$

Steady dc circuit current & voltage remain in phase

Avg or mean value of ac

$$I_{av} = \frac{1}{T/2} \int_{t_1}^{t_1 + T/2} i(t) dt \quad \text{half cycle}$$

Simple sinusoidal current $i = I_0 \sin \omega t$

$$I_{av} = \frac{1}{T/2} \int_0^{T/2} I_0 \sin \omega t \cdot dt = \frac{2 I_0}{T\omega} \left[-\cos \omega t \right]_0^{T/2} = \frac{2 I_0}{\pi} \left[\because \omega = \frac{2\pi}{T} \right]$$

$$V_{av} = \frac{2V_0}{\pi} \qquad I_{av} \cdot \frac{T}{2} = \int_{t_1}^{t_1 + T/2} i(t) dt$$

Root mean square or eff. value of ac

$$I_r = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

Heat produced by ac in a resistance R over a time period T

$$\int_0^T i^2(t) \cdot R \cdot dt = I_r^2 \cdot R \cdot T$$

for $i = I_0 \sin \omega t$

$$I_r^2 = \frac{1}{T}\int_0^T I_0^2 \sin^2 \omega t \, dt = \frac{I_0^2}{2T}\int_0^T (1 - \cos 2\omega t)\,dt$$

$$= \frac{I_0^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{I_0^2}{2}$$

Therefore, $I_r = \frac{I_0}{\sqrt{2}}$, $V_r = V_0/\sqrt{2}$

When we speak of 220V ac supply we mean its rms value. Thus, peak value of a 220V ac supply is $V_0 = \sqrt{2} \times V_r = \sqrt{2} \times 220V = 311$ V.

Form factor

The ratio of an rms value to the avg value of ac is called its form factor (K_f). For a pure sinusoidal ac,

$$K_f = \frac{I_r}{I_{av}} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = 1.11$$

Form factor gives an idea about the wave shape. Any deviation in the value of K_f from 1.11 indicates deviation from sinusoidal nature.

Peak factor

The ratio of the peak value to the rms value of any ac waveform is called its peak factor. For a pure sinusoidal ac,

$$\text{peak factor } (K_p) = \frac{I_0}{I_0/\sqrt{2}} = \sqrt{2}.$$

Power in ac circuits

$$p(t) = v(t) \cdot i(t)$$

$p(t)$ may be +Ve, 0 or −Ve.

+Ve : circuit is receiving en from the source

−Ve : source " " " " " circuit.

$$\text{avg power } P = \frac{1}{T}\int_0^T v(t) \cdot i(t)\, dt$$

$v(t) = V_0 \sin \omega t$ & $i(t) = I_0 \sin(\omega t + \phi)$

$$\therefore P = \frac{V_0 I_0}{T}\int_0^T \sin \omega t \cdot \sin(\omega t + \phi)\, dt = \frac{V_0 I_0}{2T}\int_0^T \left[\cos \phi - \cos(2\omega t + \phi)\right] dt$$

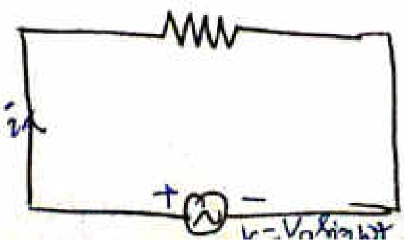
$$= \frac{V_0 I_0}{2}\cos \phi = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi = V_r \cdot I_r \cdot \underbrace{\cos \phi}_{\text{power factor}}$$

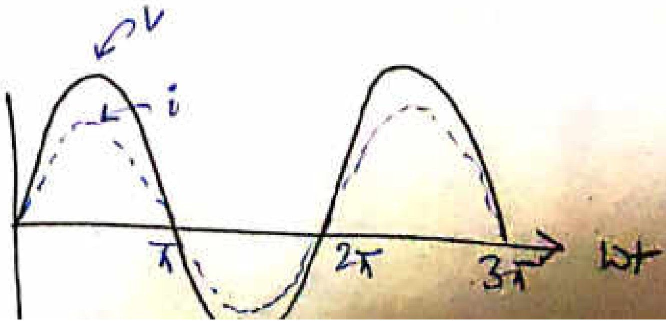
real power = apparent power × power factor

If in any case voltage and current differ in phase by 90° then $\cos \phi = 0$ and hence no power is dissipated in the circuit. The current flowing in such circuits is called wattless current.

Current flowing through a pure inductor or a pure capacitor is a wattless current.

Current – Voltage Relationship in Pure Resistance, Inductor and Capacitor





$v = Ri$ or $i = \frac{V_0}{R}\sin\omega t = I_0 \sin\omega t$

in pure resistance ~~current~~ is in phase with the app. voltage.

$$P = V_r \cdot I_r \cdot \cos\phi = \frac{V_0 I_0}{2} \cos 0^\circ = I_r^2 R.$$

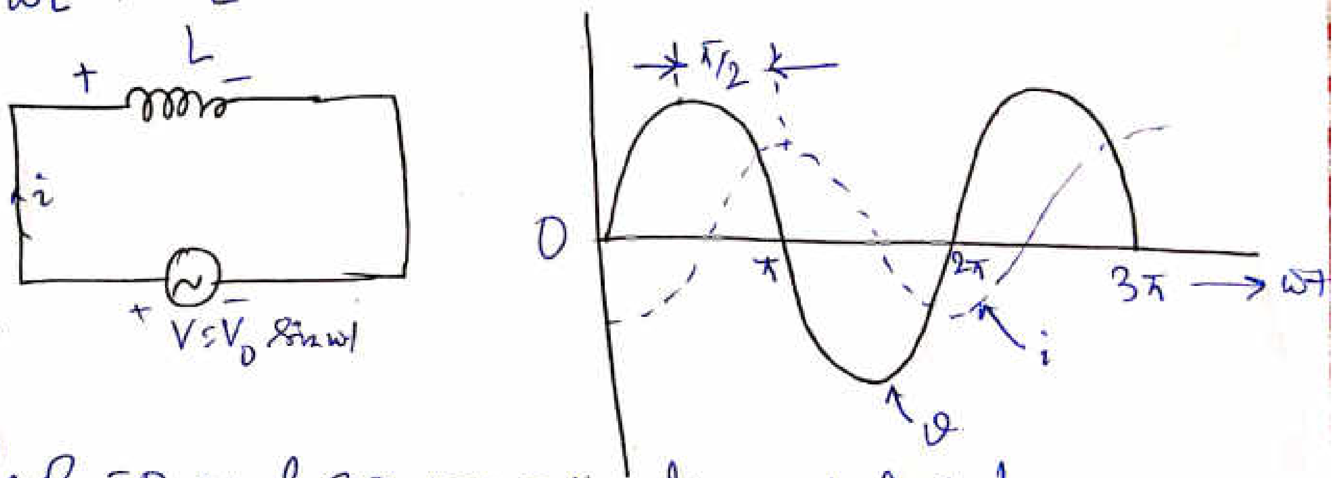
(ii) Pure inductance

$$v = V_0 \sin\omega t = L\frac{di}{dt} \quad \text{a} \quad di = \frac{V_0}{L}\sin\omega t\, dt$$

$\omega L \sim R$

$$i = \frac{V_0}{L}\int \sin\omega t\, dt = -\frac{V_0}{\omega L}\cos\omega t = -I_0 \cos\omega t = I_0 \sin(\omega t - \pi/2)$$

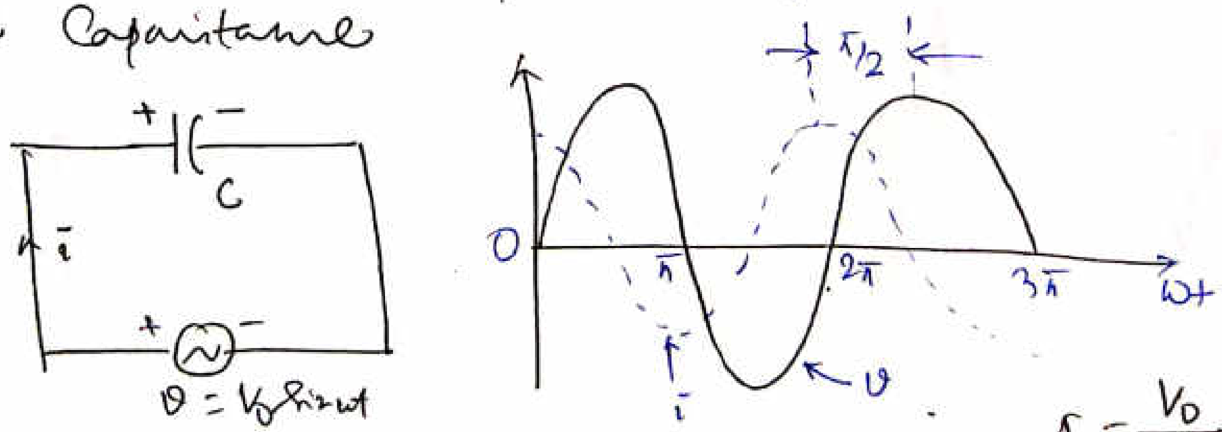
$I_0 = \frac{V_0}{\omega L}$, $\omega L \rightarrow$ inductive reactance



Choke Coil

$\cos\phi = 0$ and no power is dissipated in L.

(iii) Pure Capacitance



$I_0 = \frac{V_0}{1/\omega C}$

$$q = Cv = CV_0 \sin\omega t$$

$\omega C \sim R$

Capacitive reactance.

$$i = \frac{dq}{dt} = \omega C V_0 \cos\omega t = I_0 \sin(\omega t + \pi/2)$$

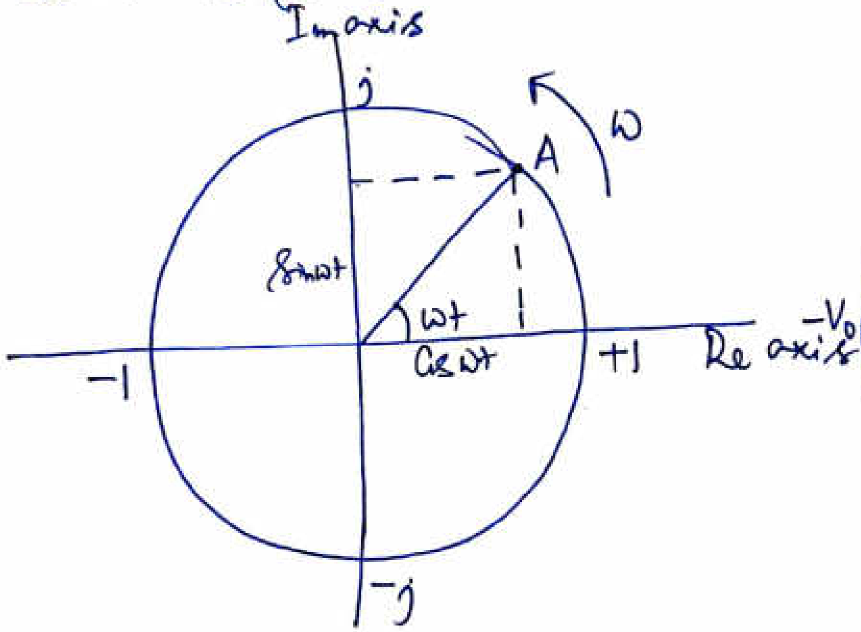
SC-path   Blocking Capacitor

Lecture Series (7)

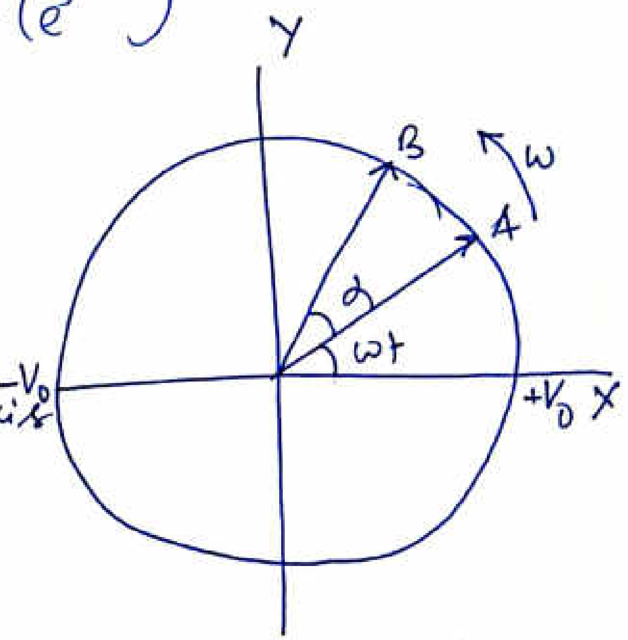
Representation of Sinusoids by Complex Numbers

$$e^{j\omega t} = \cos\omega t + j\sin\omega t \qquad (j = \sqrt{-1})$$

$$\cos\omega t = \mathrm{Re}\left(e^{j\omega t}\right) \quad \& \quad \sin\omega t = \mathrm{Im}\left(e^{j\omega t}\right)$$



$e^{j\omega t}$



phasors v & v'

Sinusoidal vol & currents can be represented in the complex plane by radius vectors having a magnitude and a phase w.r.t. a ref angle. Such radius vectors, representing complex no. are known as phasors.

$$v(t) = V_0 \sin\omega t = \mathrm{Im}\left(V_0 e^{j\omega t}\right) \qquad \vec{OA}$$

$$v'(t) = V_0 \sin(\omega t + \alpha) = \mathrm{Im}\left[V_0 e^{j(\omega t + \alpha)}\right] \qquad \vec{OB}$$

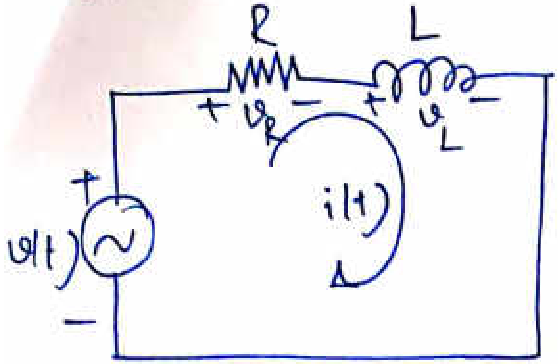
polar rep^n: $z_1 = |z_1| e^{j\theta} = a e^{j\theta} = a\angle\theta$ modulus or magnitude of z_1, θ argument, $\angle AOX = \theta$

$z_2 = z_1 e^{j\phi}$ has the same magnitude a & it leads z_1 by the angle ϕ. If $\phi = \pi/2$

then $z_2 = j z_1$, since $e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 = j$

multiplication by j means adv of phase by $\pi/2$ or 90°.

Sinusoidal Voltage Applied to a Series RL Circuit



$$Ri + L\frac{di}{dt} = v(t) \quad , \; v(t) = V_0 \cos\omega t$$

$\Downarrow$ Solⁿ of current i_r

$$Ri_r + L\frac{di_r}{dt} = V_0 \cos\omega t$$

$$Ri_i + L\frac{di_i}{dt} = V_0 \sin\omega t \quad \times j$$

$v(t) = V_0 \sin\omega t$

$\Downarrow$

i_i Solⁿ for the current

Addition (+)

Steady state Solⁿ:

$$i(t) = Ae^{j\omega t}$$

$$i = i_r + j i_i$$

$$Ri + L\frac{di}{dt} = V_0 e^{j\omega t}$$

$$(R + j\omega L)A = V_0 \Rightarrow A = \frac{V_0}{R + j\omega L} = \frac{V_0}{Z}$$

$Z (= R + j\omega L)$ impedance of the circuit, $|Z| = \sqrt{R^2 + \omega^2 L^2}$, $\tan\theta = \frac{\omega L}{R}$

$$Z = R + j\omega L = |Z| e^{j\theta}$$

$$i = i_r + j i_i = Ae^{j\omega t} = \frac{V_0}{Z} e^{j\omega t} = \frac{V_0}{|Z|} e^{j(\omega t - \theta)} \quad *$$

If $v(t) = V_0 \cos\omega t = \mathrm{Re}(V_0 e^{j\omega t})$ if $v(t) = V_0 \sin\omega t = \mathrm{Im}(V_0 e^{j\omega t})$

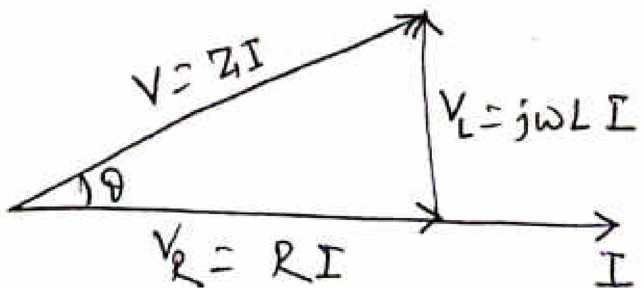
$$i_r = \frac{V_0}{|Z|} \cos(\omega t - \theta) \qquad i_i = \frac{V_0}{|Z|} \sin(\omega t - \theta)$$

Current in a series RL circuit lags the applied voltage by θ.

Reciprocal of Z is called admittance Y.

$$Y = \frac{1}{Z} = \frac{1}{R + j\omega L}$$

Putting $I_0 = \frac{V_0}{|Z|}$ $\quad * \quad \frac{V_0}{|Z| e^{j\theta}} = I_0 e^{-j\theta}$

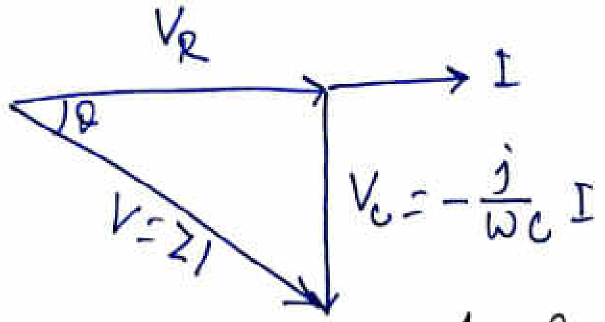


$|V| = V_0/\sqrt{2}$ and $|I| = I_0/\sqrt{2}$

$$\frac{|V|e^{j0}}{|Z|e^{j\theta}} = |I|e^{-j\theta} \quad \text{or} \quad \frac{V}{Z} = I$$

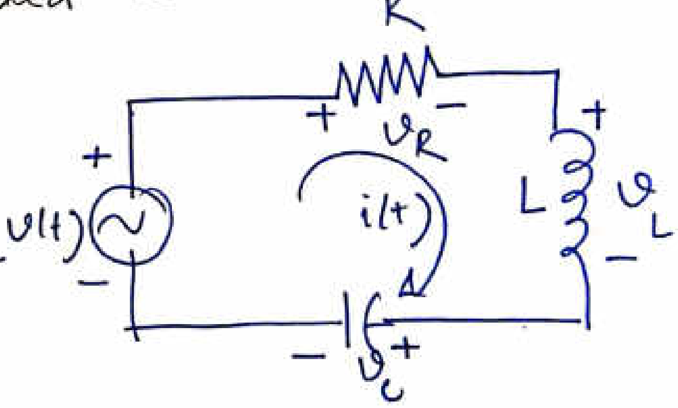
phasor eq^n rep^m a gen. Ohm's law

Homework: Sinusoidal Vol app to a Series RC Circuit



$Z [= R - \frac{j}{\omega c}]$

Sinusoidal Voltage Applied to a Series RLC Circuit



$$Ri + L\frac{di}{dt} + \frac{q}{C} = V_0 e^{j\omega t}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = j\omega V_0 e^{j\omega t}$$

$$i = Ae^{j\omega t}$$

$$\left(-\omega^2 L + j\omega R + \frac{1}{C}\right)A = j\omega V_0$$

$$\left(R + j\omega L + \frac{1}{j\omega C}\right)A = V_0$$

Steady ~~state~~ current; $$i = \frac{V_0 e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

The impedance $$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = |Z|e^{j\theta}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

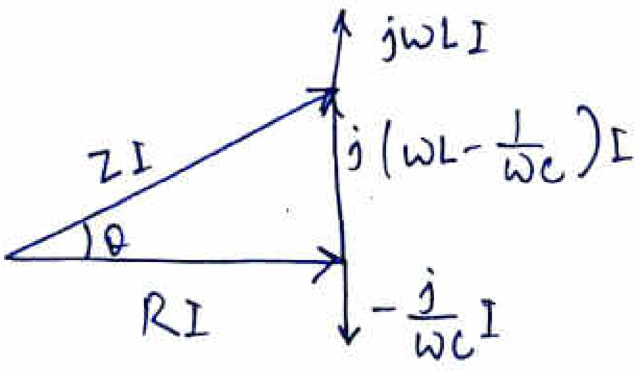
$$\tan\theta = \left(\omega L - \frac{1}{\omega C}\right)/R = \frac{X}{R} \quad . \; X\left[= \omega L - 1/(\omega C)\right]$$

$$i = \frac{V_0}{|Z|} e^{j(\omega t-\theta)}$$

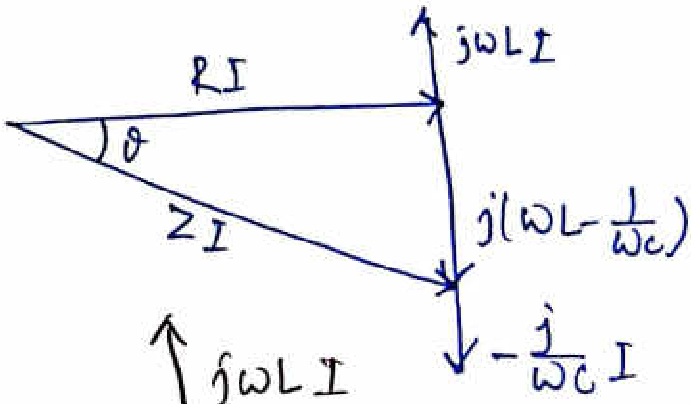
Putting $I_0 = V_0/|Z|$ and using r.m.s. values $|I| = I_0/\sqrt{2}$ & $|V| = V_0/\sqrt{2}$

$$|I| e^{-j\theta} = \frac{|V| e^{j0}}{|Z| e^{j\theta}} \quad \text{or} \quad V = ZI$$

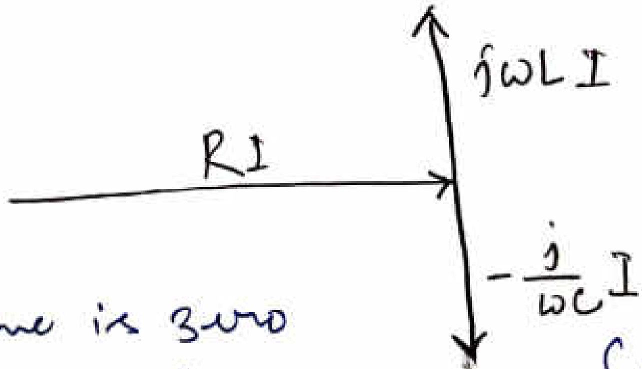
Case I: $\omega L > \frac{1}{\omega C}$



Case II: $\omega L < \frac{1}{\omega C}$



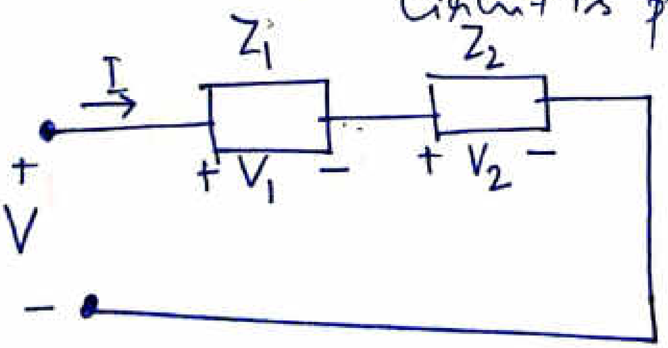
Case III: $\omega L = \frac{1}{\omega C}$



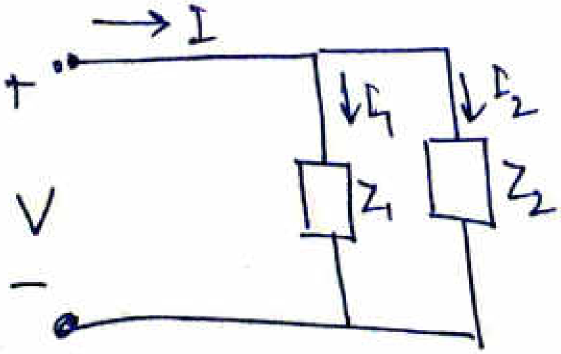
reactance is zero

Circuit is purely resistive.

Series resonance



$$Z = Z_1 + Z_2 + Z_3 + \dots + Z_n = \sum_{i=1}^{n} Z_i$$



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n} = \sum_{i=1}^{n} \frac{1}{Z_i}$$

$$Y = Y_1 + Y_2 + \dots + Y_n = \sum_{i=1}^{n} Y_i$$

Series Resonance

The impedance of a series RLC circuit

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The condn for resonance, setting reactance term to zero

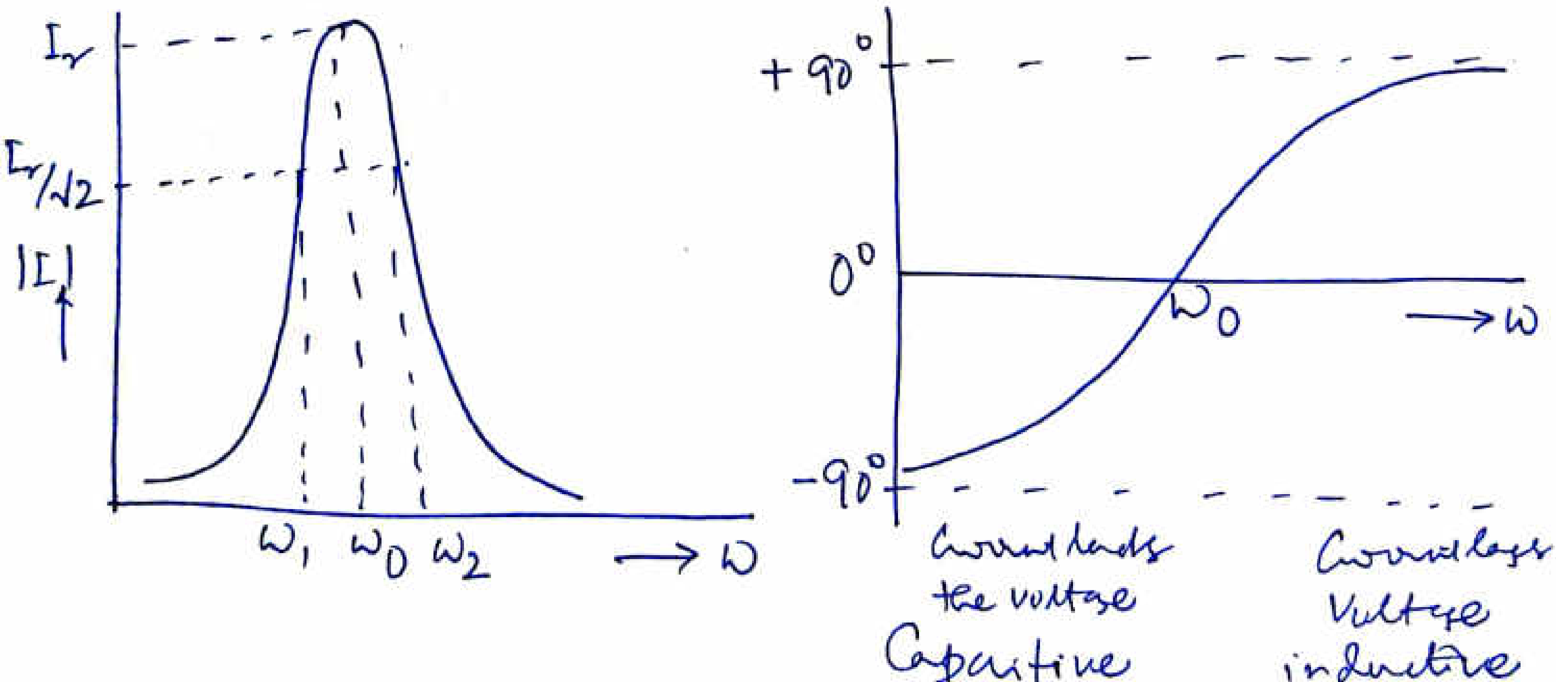
$$\omega L - \frac{1}{\omega C} = 0 \quad \text{or} \quad \omega = \omega_0 = \frac{1}{\sqrt{LC}}.$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

The impedance at resonance is R, while at off-resonant freq the impedance has the magnitude

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

As the current response of the series circuit is maxm at resonance, the circuit is sometimes called an acceptor circuit.



$\omega \approx \omega_0$, $\theta = 0$, resistive