Lecture Notes - 4

Resistance & Capacitance

$$R = \frac{\rho_c l}{S}$$

$$R = \frac{V}{I} = \frac{\int_V \vec{E}\cdot d\vec{l}}{\int_S \sigma \vec{E}\cdot d\vec{S}}$$

$$P = I^2 R$$

The resistance R (or conductance $G = \frac{1}{R}$) of a given conducting material can be found by following steps:

1. Choose a suitable coord system
2. Assume V_0 as the pot diff betn the conr terminal
3. Solve Laplace's eqn $\nabla^2 V$ to obtain V then determine $\vec{E}$ from $\vec{E} = -\vec{\nabla} V$ and find I from $I = \int \sigma \vec{E}\cdot d\vec{S}$
4. Finally, obtain R as V_0/I.

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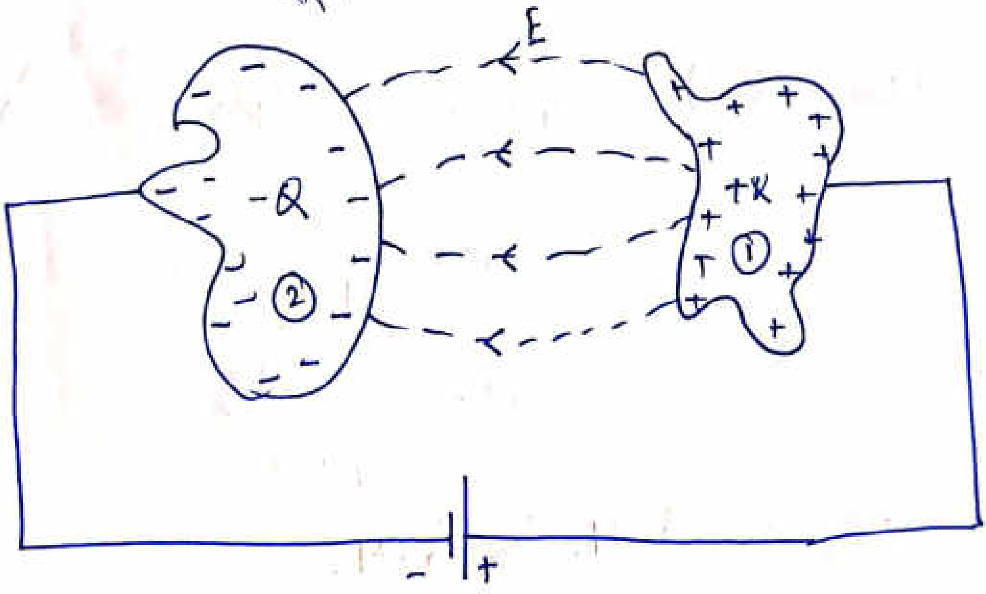
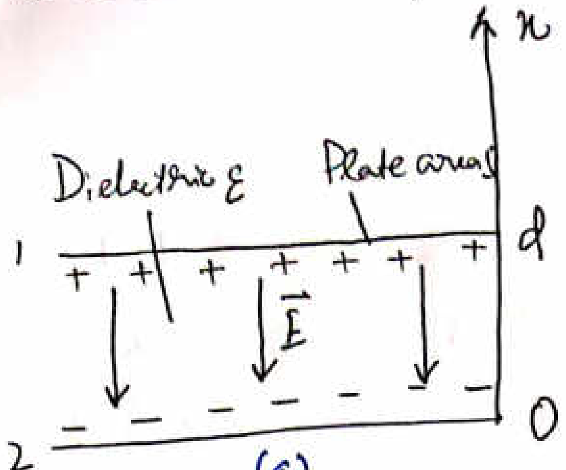


$$V = V_1 - V_2 = -\int_2^1 \vec{E}\cdot d\vec{l}$$

$$C = \frac{Q}{V} = \frac{\varepsilon \oint \vec{E}\cdot d\vec{S}}{\int \vec{E}\cdot d\vec{l}}$$

Parallel - Plate Capacitor



(a)

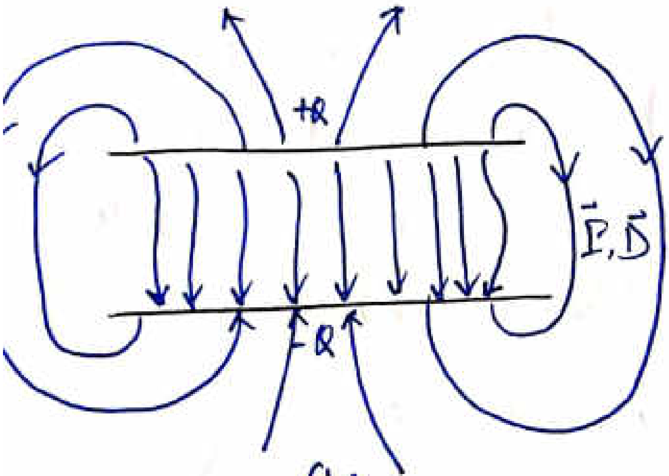
$$\rho_s = \frac{Q}{S} \qquad \vec{D} = -\rho_s\, \hat{a}_x$$

$$\vec{E} = \frac{\rho_s}{\varepsilon}\left(-\hat{a}_x\right)$$

$$= -\frac{Q}{\varepsilon S}\hat{a}_x \ *$$

$$V = -\int_2^1 \vec{E}\cdot d\vec{l} = -\int_0^d \left[-\frac{Q}{\varepsilon S}\hat{a}_x\right] dx\,\hat{a}_x$$

$$= \frac{Qd}{\varepsilon S}$$



(b)

and thus for a parallel-plate Capacitor

$$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$$

$$\varepsilon_r = \frac{C}{C_0}$$

en stored

$$W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

$$W_E = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\,dv = \frac{1}{2}\int_V \varepsilon_0 E^2\,dv$$

$$* \ W_E = \frac{1}{2}\int_V \varepsilon\,\frac{Q^2}{\varepsilon^2 S^2}\,dv = \frac{\varepsilon Q^2 S d}{2\varepsilon^2 S^2}$$

$$= \frac{Q^2}{2}\left(\frac{d}{\varepsilon S}\right) = \frac{Q^2}{2C}$$

$$= \frac{1}{2}QV$$

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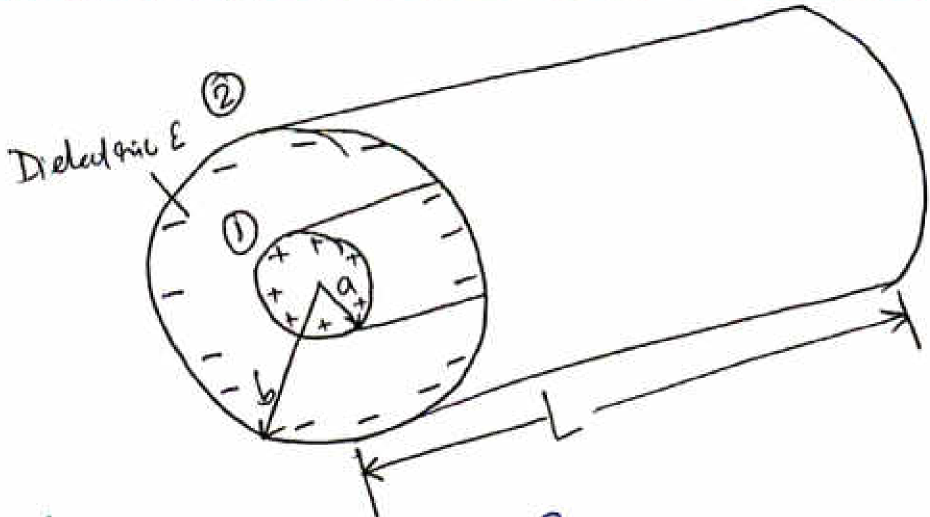
Coaxial Capacitor



By applying Gauss's law to an arb. Gaussian cylindrical surface ρ $(a<\rho<b)$ we obtain

$$Q = \varepsilon \oint \vec{E} \cdot d\vec{s} = \varepsilon E_\rho 2\pi\rho L$$

Hence: $$E = \frac{Q}{2\pi\varepsilon\rho L}\hat{a}_\rho$$

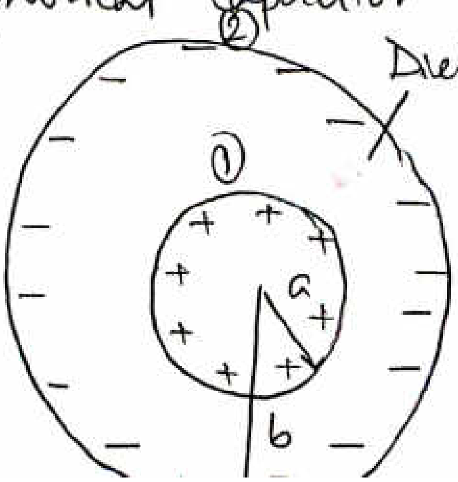
$$V = -\int_2^1 \vec{E} \cdot d\vec{l} = -\int_b^a \left[\frac{Q}{2\pi\varepsilon\rho L}\hat{a}_\rho\right] \cdot d\rho\, \hat{a}_\rho$$

$$= \frac{Q}{2\pi\varepsilon L}\ln\frac{b}{a}$$

Thus the Capacitance of a coaxial cylinder is given b

$$\boxed{C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}}$$

Spherical Capacitor



Gaussian spherical surface of radius r $(a<r<b)$

$$Q = \varepsilon \oint \vec{E} \cdot d\vec{s} = \varepsilon E_r 4\pi r^2$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r$$

$$V = -\int_2^1 \vec{E}\cdot d\vec{l} = -\int_b^a \left[\frac{Q}{4\pi\varepsilon r^2}\hat{a}_r\right]\cdot dr\,\hat{a}_r$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Thus the capacitance of the spherical capacitor is

$$\boxed{C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}}$$

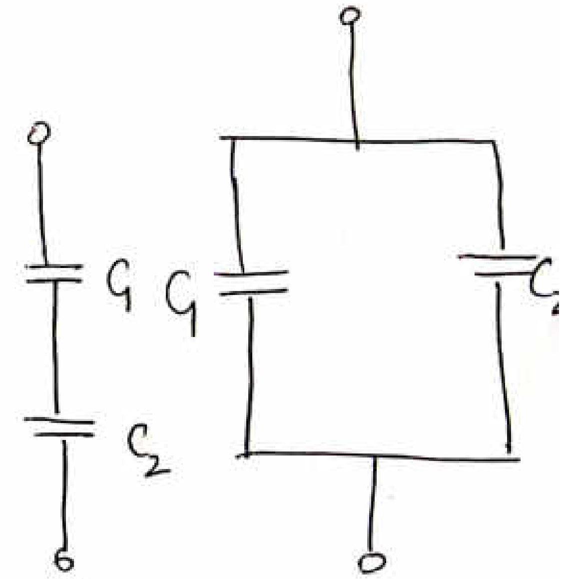
by letting $b \to \infty$, $C = 4\pi\varepsilon a$

$$R = \frac{V}{I} = \frac{\int \vec{E}\cdot d\vec{l}}{\oint \sigma\vec{E}\cdot d\vec{S}}$$

$$C = \frac{Q}{V} = \frac{\varepsilon\oint \vec{E}\cdot d\vec{S}}{\int \vec{E}\cdot d\vec{l}}$$

$$RC = \frac{\varepsilon}{\sigma}$$

which is the relaxation time T_r of the medium separating the conductors.



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$C = C_1 + C_2$$

parallel plate capacitor

$$C = \frac{\varepsilon S}{d}, \quad R = \frac{d}{\sigma S}$$

Isolated Sph. conductor

$$C = 4\pi\varepsilon a$$

$$R = \frac{1}{4\pi\sigma a}$$

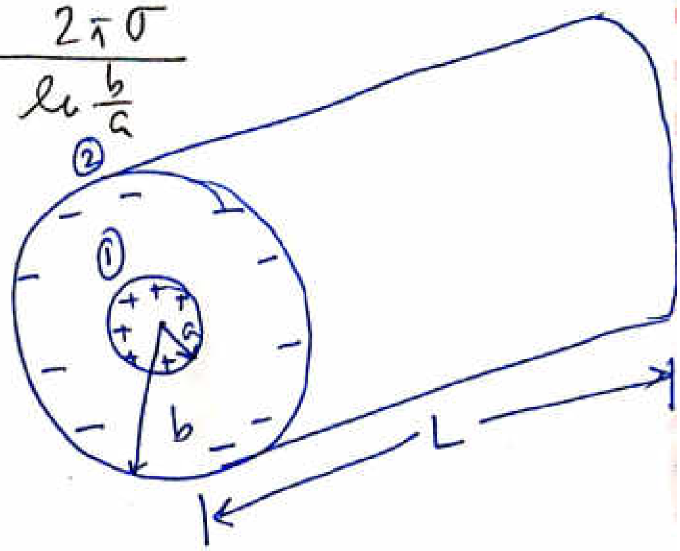
Cylindrical capacitor

$$C = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}, \quad R = \frac{\ln\frac{b}{a}}{2\pi\sigma L}$$

Sph. Capacitor

$$C = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}, \quad R = \frac{\frac{1}{a} - \frac{1}{b}}{4\pi\sigma}$$

Ex. A Coaxial cable contains an insulating material of conductivity σ. If the radius of the central wire is a and that of the sheath is b, show that the conductance of the cable per unit length is $G = \dfrac{2\pi\sigma}{\ln\frac{b}{a}}$



Sol^n: Let V_0 be the pot. diff^{ce} bet^n the inner & outer cond^{rs}

$$V(\rho=a)=0 \text{ and } V(\rho=b)=V_0$$

$$\vec{J}=\sigma\vec{E}=\frac{-\sigma V_0}{\rho\ln\frac{b}{a}}\hat{a}_\rho$$

$$d\vec{S}=-\rho\, d\phi\, dz\, \hat{a}_\rho$$

$$I=\int\vec{J}\cdot d\vec{S}=\int_{\phi=0}^{2\pi}\int_{z=0}^{L}\frac{V_0\sigma}{\rho\ln\frac{b}{a}}\rho\, dz\, d\phi$$

$$=\frac{2\pi L\sigma V_0}{\ln\frac{b}{a}}$$

The resistance per unit length is

$$R=\frac{V_0}{I}\cdot\frac{1}{L}=\frac{\ln\frac{b}{a}}{2\pi\sigma}$$

and the conductance per unit length is

$$G=\frac{1}{R}=\frac{2\pi\sigma}{\ln\frac{b}{a}} \quad \text{as required.}$$