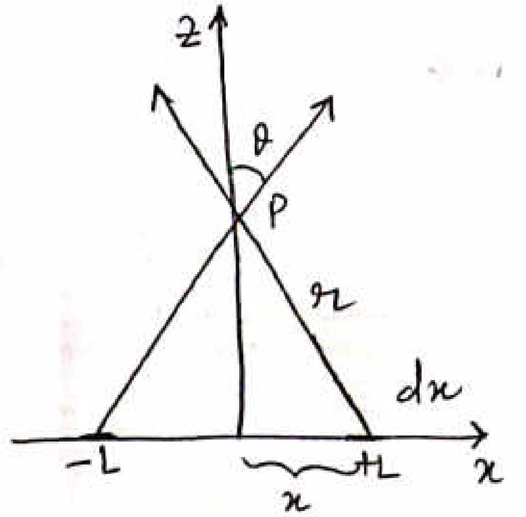
Sem-II

Lecture Notes-2



Sol^n

$$d\vec{E} = 2\,\frac{1}{4\pi\epsilon_0}\left(\frac{\lambda\,dx}{r^2}\right)\cos\theta\;\hat{z}$$

$$\cos\theta = \frac{z}{r},\quad r = \sqrt{z^2+x^2}$$

$x \to 0$ to L

$$E = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{2\lambda z}{(z^2+x^2)^{3/2}}\,dx$$

$$= \frac{2\lambda z}{4\pi\epsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]\Bigg|_0^L$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{2\lambda L}{z\sqrt{z^2+L^2}}\quad \uparrow z$$

Horizontal components of the two fields cancel

$z \gg L$

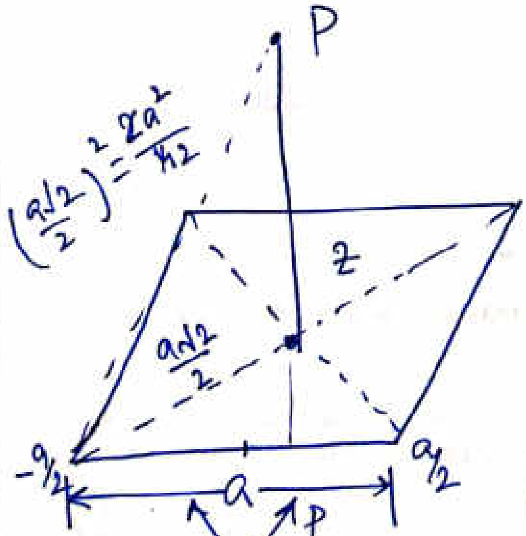
$$E \simeq \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z^2}$$

pt. charge $q = 2\lambda L$

$$E \sim \frac{q}{4\pi\epsilon_0 z^2} \qquad L\to\infty$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{z} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{s}$$

$s \to$ distance from the wire.



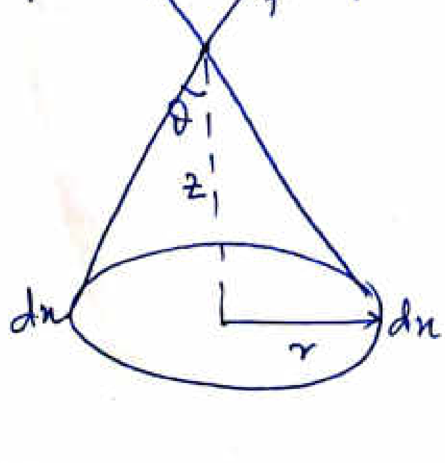
$\cancel{q = 2\lambda L,\ L\to\infty}$

$\cancel{E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{z}}$

$\cancel{E' = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{r\sqrt{r^2+L^2}}}$

$L \to \frac{a}{2}$

$z \to \sqrt{z^2+\left(\frac{a}{2}\right)^2}$



(distance from centre of edge to P) field of one edge is:

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{\lambda a}{\sqrt{z^2+\frac{a^2}{4}}\sqrt{z^2+\frac{a^2}{4}+\frac{a^2}{4}}}$$

vertical component, $4\cos\theta = 4\frac{z}{\sqrt{z^2+a^2/4}}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{4\lambda a z}{\left(z^2+\frac{a^2}{4}\right)\sqrt{z^2+a^2/2}}\;\hat{z}$$

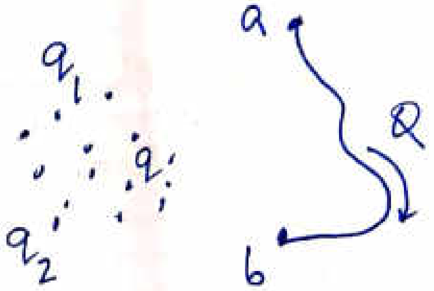
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left\{\int \frac{\lambda\, dl}{\mathscr{r}^2}\cos\theta\right\}\hat{z}$$

$$\mathscr{r}^2 = r^2 + z^2, \quad \cos\theta = \frac{z}{\mathscr{r}} \qquad \int dl = 2\pi r$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda(2\pi r)z}{(r^2+z^2)^{3/2}}\hat{z}$$

Work and Energy in Electrostatics

The Work Done to Move a charge



$$\vec{F} = Q\vec{E}$$

$$W = \int_a^b \vec{F}\cdot d\vec{l} = -Q\int_a^b \vec{E}\cdot d\vec{l}$$

$$V(\vec{b}) - V(\vec{a}) = \frac{W}{Q} \qquad = Q\left[V(\vec{b}) - V(\vec{a})\right]$$

indep of path

el force "Conservative"

bring the charge Q in from far away and stick it at $\vec{r}$

$$W = Q\left[V(\vec{r}) - \underbrace{V(\infty)}_{\text{ref pt.}}\right]$$

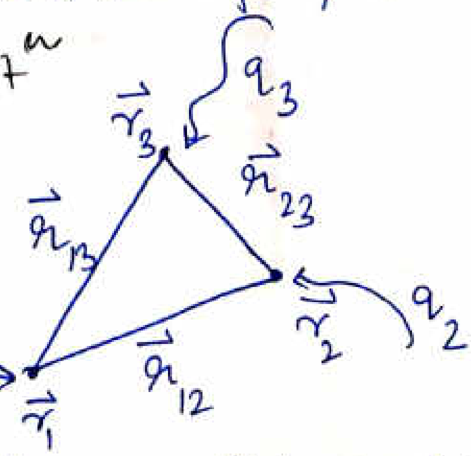
$$= Q\,V(\vec{r})$$

'potential' is pot en/Unit charge (field is the force/Unit charge)

The Energy of a pt. Charge Dist^n

$\xrightarrow{\text{bring}}$ q_2 $\qquad q_2 V_1(\vec{r}_2)$

V_1 is the pot due to q_1



$$W_2 = \frac{1}{4\pi\epsilon_0} q_2 \left(\frac{q_1}{\mathscr{r}_{12}}\right)$$

q_1 $\underline{1st}$ no work $\qquad$ no fld yet to fight

$q_3 \longrightarrow$ Work $q_3 V_{1,2}(\vec{r}_3)$ pot due to q_1 & q_2

$$W_3 = \frac{1}{4\pi\varepsilon_0} q_3 \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$$

Similarly $W_4 = \frac{1}{4\pi\varepsilon_0} q_4 \left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}} \right)$

total work $W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}} \right)$

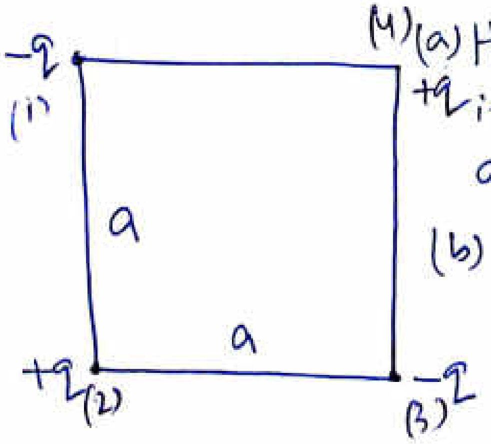
$$W = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j>i}}^{n} \frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{8\pi\varepsilon_0} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_i q_j}{r_{ij}}$$

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{1}{4\pi\varepsilon_0} \frac{q_j}{r_{ij}} \right)$$

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\vec{r}_i)$$ Configuration of pt. charges

2.31



(a) How much work does it take to bring in another charge, +q from far away and place it in the fourth corner?

(b) How much work does it take to assemble the whole conf^n of four charges?

Scanned by CamScanner

(a) $V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_{ij}} \cdot \frac{1}{4\pi\varepsilon_0}\left\{-\frac{q}{a} + \frac{q}{\sqrt{2}a} + \frac{-q}{a}\right\}$

$$= \frac{q}{4\pi\varepsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

$$\therefore W_4 = qV = \frac{q^2}{4\pi\varepsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

(b) $W_1 = 0.$ $W_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q^2}{a}\right)$; $W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right)$

$$W_4 = \frac{q^2}{4\pi\varepsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

$$W_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left\{-1 + \frac{1}{\sqrt{2}} - 1 - 2 + \frac{1}{\sqrt{2}}\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

The Energy of a Continuous Charge Distr^n

$$W = \frac{1}{2}\int \rho V \, d\tau$$

$\rho = \varepsilon_0 \vec{\nabla}\cdot\vec{E}$, so $W = \frac{\varepsilon_0}{2}\int (\vec{\nabla}\cdot\vec{E}) V \, d\tau$

int^n by parts

$$W = \frac{\varepsilon_0}{2}\left[-\int \vec{E}\cdot(\vec{\nabla} V)\, d\tau + \oint V\vec{E}\cdot d\vec{a}\right].$$

$$= \frac{\varepsilon_0}{2}\left(\int_V E^2 \, d\tau + \oint_S V\vec{E}\cdot d\vec{a}\right)$$

at large distances, $E \sim \frac{1}{r^2}$, $V \sim \frac{1}{r}$, $d\vec{a} \sim r^2$, $\int \to \frac{1}{r}$

$$W = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau$$

2.8
Find the en of a uniformly charged sph. shell of total charge q and radius R.

Solⁿ 1 — Surface charge is. $W = \frac{1}{2}\int \sigma V\, da$.

pot. at the surface of this sphere is $\left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{R}$ (const)

$$W = \frac{1}{8\pi\epsilon_0}\frac{q}{R}\int \sigma\, da = \frac{1}{8\pi\epsilon_0}\frac{q^2}{R}.$$

Solⁿ 2 — $\vec{E} = 0$; outside,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}, \quad \text{So } E^2 = \frac{q^2}{(4\pi\epsilon_0)^2 r^4}$$

$$W_{tot} = \frac{\epsilon_0}{2(4\pi\epsilon_0)^2}\int_{\text{outside}}\left(\frac{q^2}{r^4}\right)\left(r^2 \sin\theta\, dr\, d\theta\, d\phi\right)$$

$$= \frac{1}{32\pi^2\epsilon_0}q^2\, 4\pi\int_R^\infty \frac{1}{r^2}\,dr = \frac{1}{8\pi\epsilon_0}\frac{q^2}{R}.$$

2.32 Find the en stored in a uniformly charged solid sphere of radius R and charge q. Do it three diff^nt ways:

(a) Use $W = \frac{1}{2}\int \rho V\, d\tau$ $\quad \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}$

(b) $$W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2\, d\tau$$

(c) $$W = \frac{\epsilon_0}{2}\left(\int_V E^2\, d\tau + \oint_S V\vec{E}\cdot d\vec{a}\right)$$

Take a sph vol of radius a. Notice what happens at $a \to \infty$.