Sem-II

Lecture Notes-1

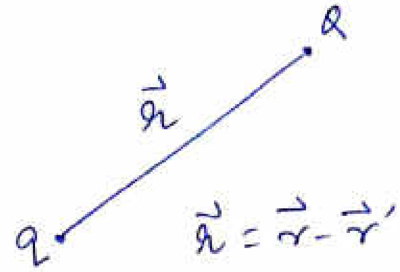
Introductory Electrostatics

Source charges

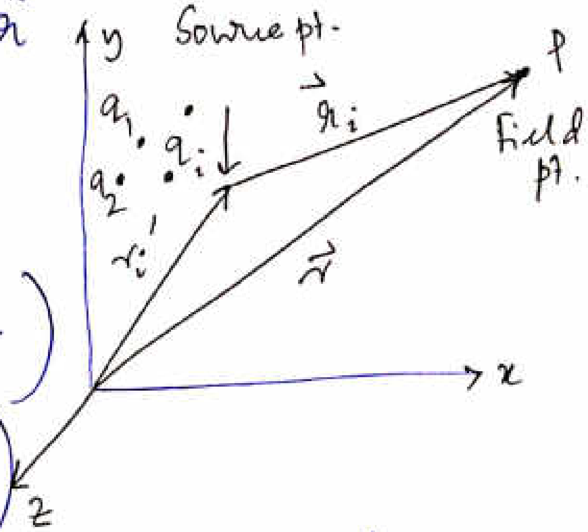
Test charge

principle of superposition

Coulomb's Law



$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}}$$



Electric field

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$

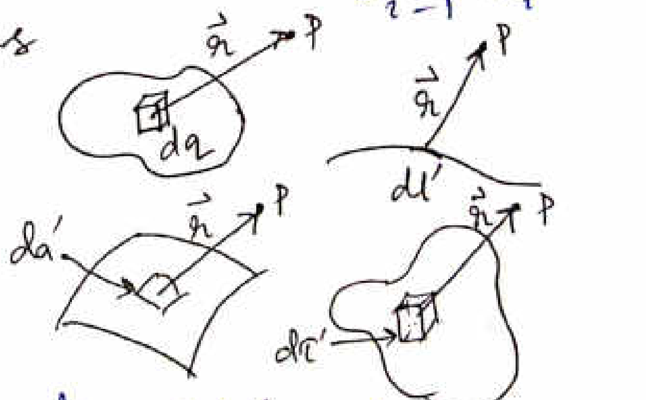
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{\mathscr{r}_1^2}\hat{\mathscr{r}}_1 + \frac{q_2 Q}{\mathscr{r}_2^2}\hat{\mathscr{r}}_2 + \cdots\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1\hat{\mathscr{r}}_1}{\mathscr{r}_1^2} + \frac{q_2\hat{\mathscr{r}}_2}{\mathscr{r}_2^2} + \cdots\right)$$

or $\vec{F} = Q\vec{E}$ where $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i$

Continuous Charge Distributions

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,dq$$



$dq = \sigma da'$ $\quad$ $dq = \rho d\tau'$

due to line charge $dq = \lambda dl'$

$$E(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_P\frac{\lambda(\vec{r}')}{\mathscr{r}^2}\hat{\mathscr{r}}\,dl'$$

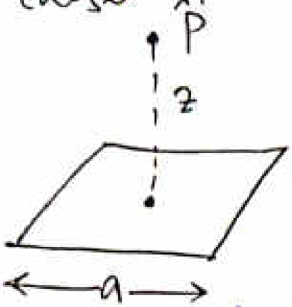
due to surface charge

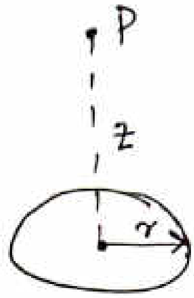
$$E(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_S\frac{\sigma(\vec{r}')}{\mathscr{r}^2}\hat{\mathscr{r}}\,da'$$

due to vol. charge

$$E(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V\frac{\rho(\vec{r}')}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau'$$

Exercise : ① Find the el fld a distance z above the midpt. of a st line segment of length $2L$, which carries a uniform line charge λ.

② 

③ 

Find the el fld a distance z above the centre of a sq loop (side a) carrying uniform line charge λ

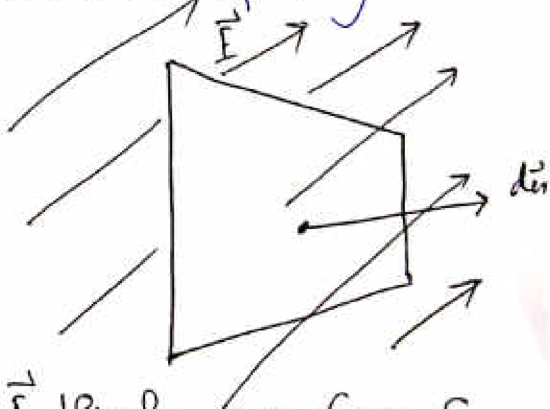
Find the el fld

Gauss's law

In the case of a pt. charge q at the origin, the flux of $\vec{E}$ through a sphere of radius r is

$$\oint \vec{E}\cdot d\vec{a} = \int \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{r}\right)\cdot\left(r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}\right)$$

$$= \frac{1}{\epsilon_0} q$$



$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i$$

$$\oint \vec{E}\cdot d\vec{a} = \sum_{i=1}^{n}\left(\oint \vec{E}_i \cdot d\vec{a}\right) = \sum_{i=1}^{n}\left(\frac{1}{\epsilon_0} q_i\right)$$

flux of $\vec{E}$ through a surface S,

$$\Phi_E \equiv \int_S \vec{E}\cdot d\vec{a}$$

For any closed surface

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc}.$$

applying div th :

$$\oint_S \vec{E}\cdot d\vec{a} = \int_V (\vec{\nabla}\cdot\vec{E})\, d\tau \Rightarrow \int_V (\vec{\nabla}\cdot\vec{E})\, d\tau = \int_V \left(\frac{\rho}{\epsilon_0}\right) d\tau$$

again $Q_{enc} = \int_V \rho\, d\tau$ ②

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Conservation Laws

The Continuity Eq^n

global charge (local)

If the total charge in some volume changes, then exactly that amt of charge must have passed in or out through the surface.

Charge in a vol V

$$Q(t) = \int_V \rho(\vec{r}, t)\, d\tau$$

current flowing out through the surface S is $\int_S \vec{J}.d\vec{a}$

local Conser of charge

$$\frac{dQ}{dt} = -\int_S \vec{J}.d\vec{a}$$

div th

$$\int_V \frac{\partial \rho}{\partial t}\, d\tau = -\int_V (\vec{\nabla}.\vec{J})\, d\tau$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\vec{\nabla}.\vec{J}}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = -\vec{\nabla} V$$

Electrodynamics Before Maxwell

(i) $\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0}$ (Gauss's law)

(ii) $\vec{\nabla}.\vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's law)

(iv) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ (Ampere's law)

div of Curl is always zero

$\vec{\nabla}.(\vec{\nabla} \times \vec{B}) = \mu_0(\vec{\nabla}.\vec{J})$ For steady current, the div $\vec{J}$ is zero applying Continuity eq^n and Gauss's law

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}\left(\epsilon_0 \vec{\nabla}\cdot\vec{E}\right) = -\vec{\nabla}\cdot\left(\epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

Combine $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ with $\vec{J}$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

when $\vec{E}$ is Const, $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$

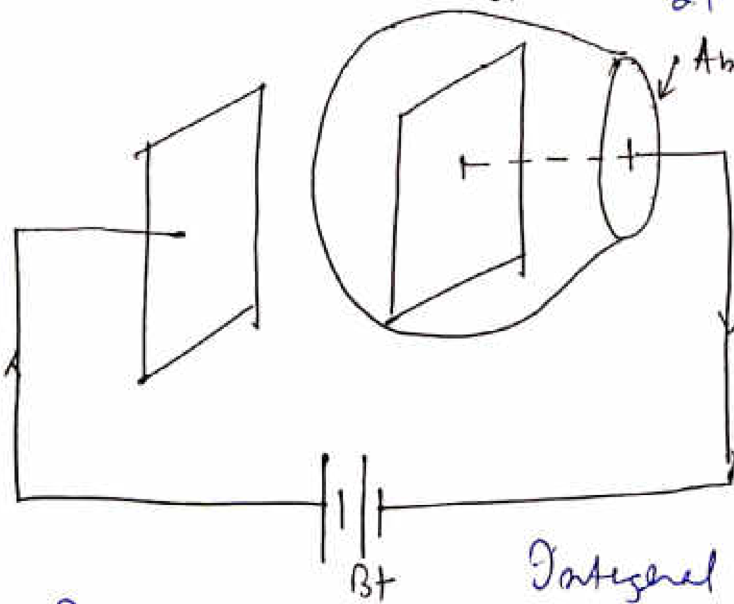
Just as a changing mag fld induces an el fld (Faraday's law) so

A changing el fld induces a mag fld.

1888

Displacement current

$$\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



Integral form of Ampere's law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

I_{enc} ?

$$E = \frac{1}{\epsilon_0}\sigma = \frac{1}{\epsilon_0}\frac{Q}{A}$$

bet^n the plates $\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0 A}\frac{dQ}{dt} = \frac{1}{\epsilon_0 A} I$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \int\left(\frac{\partial \vec{E}}{\partial t}\right)\cdot d\vec{a}$$

(A)

flat surface $E = 0$ & $I_{enc} = I$

ballon shaped surface