

Bhairab Ganguly College

4th SEM, Class Test-2020
Paper-CORE T8, Mathematical Physics
Class Teacher: Dr. D. Bhadra

Time: 2 hour

Full Marks: 40

1. Answer any **five** from the following

2X5=10

- i) What is the Fourier cosine transform of $e^{-|x|}$?
- ii) What is the Fourier transform of the convolution of $F(t)$ with $\delta(t-t_0)$?
- iii) Prove that $(\det(e^A) = \exp(\text{Tr}A))$.
- iv) Diagonalise $\begin{vmatrix} \alpha & i\beta \\ -i\beta & \alpha \end{vmatrix}$
- v) If a unitary matrix U be written as $U=A+iB$, A & B being Hermitian and having non-degenerate eigen-values. Find out the condition satisfied by A & B .
- vi) The eigen-values of a matrix are 1 , $\frac{1}{\sqrt{2}}(1+i)$ and $\frac{1}{\sqrt{2}}(1-i)$ respectively. The matrix is?
- vii) For what values of α , the matrix $\begin{pmatrix} \alpha & 1 \\ 1 & 0 \end{pmatrix}$ is invertible.
- viii) Prove that an odd dimensional matrix with real elements has atleast one real eigen-value.

Answer any **three** from the following.

3X10=30

Q2. (a) Find the F.T. of

(i) $f(t) = 1$ for $t > 0$ & -1 for $t < 0$, and vanishes for $|t| > 1$

(ii) $f(t) = v(t) \cos \omega_0 t$ 3

(b) Prove that the diagonalising matrix of a unitary matrix is unitary. 3

(c) The matrix A is given by $A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 5 \end{vmatrix}$. Find out the eigen-value of

$A^3 + 4A^2 + 3A + 2I$.

Q3. (a) The matrix A is given by $\begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix}$ in the standard basis $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Find A

with respect to the basis $\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ -7 \end{pmatrix}$ 4

(b) Diagonalise the matrix $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$ 4

(c) A 3X3 matrix satisfies the equation $M^2 - 3M + 2I = 0$ (I is the identity matrix). Find out the determinant of the matrix if its trace is 6. 2

Q4. (a) Solve $x_1 + x_2 - x_3 - x_4 = 3$
 $x_1 - x_2 + x_3 - x_4 = 1$ 3

(b) If $\begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} u \\ v \end{pmatrix}$ are linearly independent write $\begin{pmatrix} a \\ b \end{pmatrix}$ in terms of $\begin{pmatrix} x \\ y \end{pmatrix}$ and $\begin{pmatrix} u \\ v \end{pmatrix}$. 3

(c) Are the matrix $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ linearly independent? If they are not orthogonal construct an orthogonal set of vectors out of them. 4

Q5. (a) Find the Fourier cosine transform of $e^{-\alpha|x|}, e^{-\alpha x^2}$ 4

(b) What is the F.T. of the equation $\frac{d^2 y}{dx^2} + f(x)y = 0$? 2

(c) Solve by F.T. the differential equation of a damped S.H.O. subjected to an external periodic force $F \sin \omega t$. 4

(Hint: The equation of motion is $m \frac{d^2 x}{dt^2} + k \frac{dx}{dt} + x(t) = F \sin \omega t$ Take the F.T. of both sides.)